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SOIL SHEAR MODULUS FROM RESONANT COLUMN, TORSIONAL SHEAR AND BENDER ELEMENT TESTS

Zsolt Szilvágyi[1], Péter Hudacsek[1] and Richard P. Ray[1]

[1]Department of Structural and Geotechnical Engineering, Széchenyi István University, Hungary

ABSTRACT: This study compares results from three different testing methods: Resonant Column, Torsional Simple Shear, and Bender Element tests to determine shear modulus. The resonant column and torsional shear tests were performed on the same hollow cylinder specimen. The bender element test was performed on a triaxial specimen with the same void ratio and confining stress as well as others. Several effects were studied, among them confining stress, shear strain amplitude and for the bender element, anisotropic confinement. Testing methods and data analysis are discussed in the paper because data interpretation is very important in these tests. Results showed that the shear modulus values were almost identical between the resonant column and torsional shear but varied somewhat with the bender element results. Further research will focus on influence of stress anisotropy preparation methods.

Keywords: Shear modulus, Resonant column, Torsional shear, Bender element, Small strain stiffness

1. INTRODUCTION

Soil modulus has been used to model the low and medium stress-strain behavior of soil since the development of soil mechanics. Using elasticity theory and judiciously selected values for modulus and Poisson's ratio, geotechnical engineers have estimated foundation settlement, pile load deflection in both vertical and horizontal directions and a wide variety of other applications. One of the difficulties with such an approach is the inability to make accurate and repeatable measurements of soil modulus. Measuring Young's modulus E, or shear modulus, G in the field or laboratory is a challenge [1,2]. For most applications, the measured modulus is back calculated from a fairly complicated test condition: a penetrating cone tip or sliding sleeve, or an expanding cylinder within a boring [3]. In the laboratory, there are also problems with precision, sample size and uniform stress or strain state. While certain properties such as void ratio, water content, or density can be faithfully repeated, fabric, cementation, and particle orientation cannot. One of the difficulties associated with laboratory tests is achieving a low enough strain level that will replicate field conditions. Strain values on the order of 10^{-4}% are necessary to achieve a truly low-strain elastic condition. This paper discusses measurement of shear modulus (G) at low strains (10^{-4}% to 0.1%) by three different methods: Resonant column, (RC) torsional shear (TOSS), and bender element (BE). Each method has advantages and drawbacks. The best situation would be to use all three if necessary, but how would one know they are measuring the same property? Laboratory tests on nearly identical samples of Danube sand were performed using the three methods and the results were examined to see if they could be used interchangeably.

2. RESONANT COLUMN TORSIONAL SIMPLE SHEAR (RC-TOSS) DEVICE

The RC-TOSS device can measure rotational displacement and acceleration, longitudinal displacement, torque and pore pressure [4]. These are accomplished by proximitors, an accelerometer, a linear variable differential transformer (LVDT), by coil current, and a pressure transducer, respectively. For RC tests, resonant frequency and acceleration determine modulus and strain level. For TOSS tests, the current flowing through the drive coils provide torque (stress) and proximitors measure rotation (strain) of the free end. A function generator provides the harmonic signal for RC testing while DC voltage from the data acquisition board drives the TOSS test. Both tests require a high power amplifier (500 watts) to provide higher torque levels (about 50 Nm) for nonlinear conditions.

By using an accelerometer, RC measurements benefit from the harmonic relationship between acceleration and displacement. This makes measurements at strain amplitudes near 10^{-4}% achievable. A simple computation illustrates the concept. Typical accelerometer sensitivity is 300 mV/g, and a reading of 5 mV RMS (7.07 mV peak) from a good digital meter is common. The final necessary reading is resonant frequency (ω_n).

For the configuration in Fig.1,

$$\omega_n = 375\left(\frac{rad}{\sec}\right)$$

$$a = \left(\frac{7.07}{300}\right)(9.807)(100) = 231\left(\frac{\text{cm}}{\sec^2}\right)$$

$$\delta = \left(\frac{231}{\omega_n^{\ 2}}\right) = \left(\frac{231}{375^2}\right) = 1.64x10^{-4}\left(\text{cm}\right) \quad (1)$$

$$\gamma = \delta\left(\frac{r_{avg}}{r_{accel}l}\right) = 1.64x10^{-4}\left(\frac{2.5\,\text{cm}}{4.70\,\text{cm}\;14.0\,\text{cm}}\right)$$

$$= 6.244x10^{-6}\left(\frac{\text{cm}}{\text{cm}}\right) \text{ or } 6.244x10^{-4}\%$$

Where a is acceleration, δ is displacement, γ is average shear strain, r_{ave} is average radius, r_{accel} = radial distance from center of rotation to accelerometer, and l is specimen length.

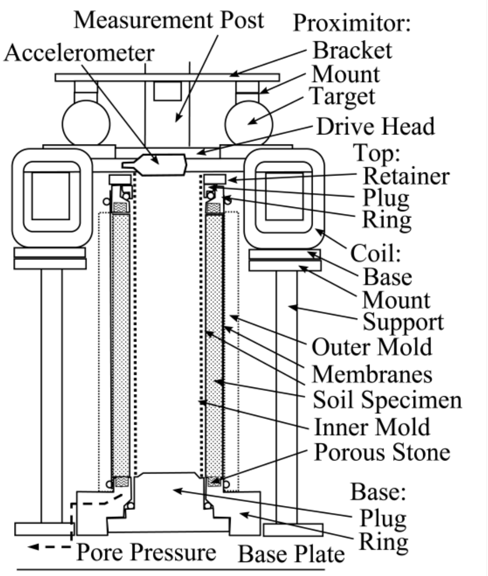

Fig.1 Cross-section of hollow cylinder sample in the RC-TOSS device

Specimen size is 4cm ID, 6cm OD and 14cm long. Magnets are free to move through coils without touching. The base is fixed and the top is free to twist as shown in Fig.1. The measurement is simple, accurate and repeatable. For TOSS tests, proximitors are used so that nothing touches the free end of the specimen. This allows for switching between TOSS and RC at any time. A proximitor measures the air gap between itself and a steel target mounted on the specimen drive head. Typical gap distance is 2.5mm and sensor repeatability allows for accuracy to about 0.001 mm. This translates to a shear strain range of about 0.01 % to 1%.

Vertical movement is monitored by an LVDT with accuracy to about 0.01mm. The height change can often indicate changes in density or dilatant/contractile behavior. Pore water volume/ pressure can also be monitored, however samples for these tests were dry or well below saturation level.

3. RESONANT COLUMN TEST METHOD

The procedure for testing the soils by Resonant Column method generally followed that outlined in ASTM D 4015-07[5]. Due to the unique nature of the material and the focus on high-amplitude behavior, the procedure deviated slightly from ASTM standards at several points listed below:

1. The specimens used were hollow cylinders. This allowed for more uniform torsional strain distribution, hence more representative values of shear modulus and damping ratio at high strain levels.
2. Platens had a sintered bronze filter stone rigidly bolted to the end caps to maintain full contact between the specimen and device.
3. The bottom end was tightly fixed to the base of the device, yielding a virtually infinite torsional stiffness at the passive end.
4. The weight of the active end was counter-balanced with a spring and thin wire, generating only negligible torsional resistance. This produced a free-end on the active side.
5. Given the fixed-free system, the governing test equation reduced to:

$$v_s = \frac{\omega_n l}{\beta} \quad (2)$$

where v_s is shear wave velocity, l is length of specimen, ω_n is resonant frequency, β is device/specimen constant, computed from device and specimen mass and geometries. It is the solution to the implicit equation

$$\frac{J}{J_0} = \beta \tan \beta \quad (3)$$

where J is mass polar moment of inertia of specimen (computed at time of testing) and J_0 is mass polar moment of inertia of device head (measured via torsional pendulum calibration).

Since all values in Eqs. (2) and (3) are known before resonating, except the natural frequency and shear wave velocity, the evaluation requires only a simple spreadsheet calculation. Damping for the RC test is evaluated using the log decrement method where amplitudes of two consecutive acceleration peaks are measured and damping is computed as

$$\delta ec = \ln(\frac{u_n}{u_{n+1}}) \quad \text{and} \quad D = \frac{\delta ec}{\sqrt{4\pi^2 + \delta ec^2}} \quad (4)$$

where δec is log decrement, u is the amplitude of acceleration of the n and $n+1$ peaks and D is damping ratio. When expressed as a percentage, the value of D is multiplied by 100%.

4. TORSIONAL SHEAR TEST METHOD

Using the same specimen dimensions, and often the same specimen as the RC test, TOSS testing is performed by driving the magnet coil system with a DC signal from the data acquisition card on the computer. A 16-bit digital-to-analog signal with peak output of ±10 volts DC drives the amplifier which then drives the coils. One step of the output corresponds to 0.0003 volts, so the resolution without any additional linear circuits is very good. The testing program is written in Visual Basic for Applications (VBA) which is free software in MS-Excel. Since the data input and output is based on spreadsheets, the VBA approach is quite direct and simple. The only complication is making sure the data acquisition system is compatible and has the proper driver software for VBA. The torsional shear test control program follows these steps:

1. Input all relevant specimen data (same as resonant column test).

2. Decide on cyclic or irregular loading history.

2a. Cyclic load is scaled by amplitude and offset. Maximum number of testing cycles is 65,000. Maximum number of cycles to record for data is 200, with 200 data points per recorded cycle.

2b. Irregular loading histories can be any arbitrary sequence up to 10,000 points. Maximum amplitude and DC offset of loading history can be set. All points are recorded as data.

3. Check the readings from proximitors, LVDT, effective confining stress and make adjustments if necessary to make sure the sensors stay within their measuring range during testing.

4. Perform test. The program looks for the next point in the load history, if the point is more positive than the present point, the program drives one step in the positive direction. If the next point is more negative, the program drives one step in the negative direction.

5. At each driving step (4), the program checks the data coming in from the sensors.

5a. For cyclic loading, if the data value reaches a data control point (one of the 200 in each cycle) then the data for applied torque, rotation, vertical deflection, and effective confining stress are recorded and the next data control point is loaded for checking.

5b. For irregular tests, all data is recorded for each point.

6. The test stops when all cycles (or irregular history points) are complete. A test of 200 cycles duration takes about five minutes to complete. An irregular history with 2000 data points requires about one minute or less.

The data is then output to the spreadsheet. Typical graphs of output for a cyclic test and irregular history test are shown in Figs.2 and 3. Specimens were confined under vacuum at 90kPa.

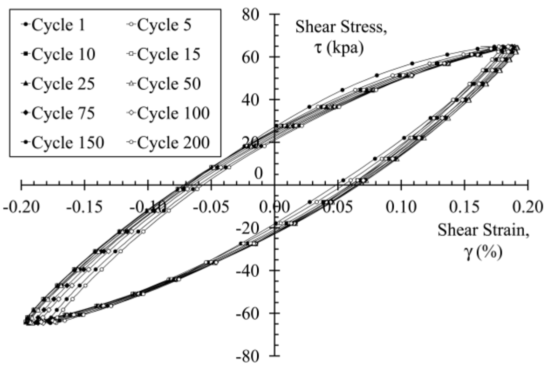

Fig.2 Cyclic test results for 200 uniform load cycles, note slight stiffening with cycles

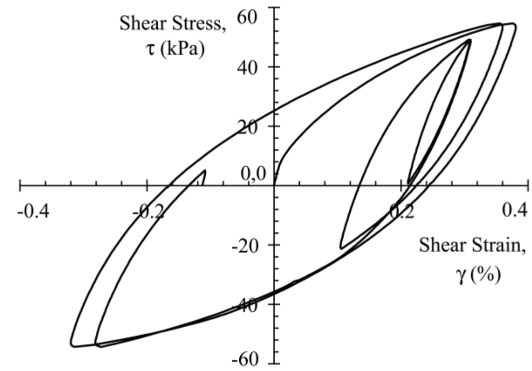

Fig.3 Irregular loading test results showing start (0,0) and several reversals

For cyclic tests, secant shear modulus, G_{sec} and hysteretic damping ratio D_{hys} can be computed as well. Secant modulus is the slope of a line connecting the two ends of any loop. Hysteretic damping is computed from the area inside the stress-strain loop, A_{loop} and the amount of work done to the specimen. For a uniform cycle of stress and strain with peak values of (±τ, ±γ),

$$D_{hys} = \frac{A_{loop}}{4\pi \frac{1}{2} \gamma \tau}$$

(5)

5. BENDER ELEMENT TRIAXIAL TEST

Bender elements were developed around the late 1980’s and have improved steadily since that time [6]. A ceramic and nickel film generates motion when driven by an oscillating electrical signal (Fig.4). When buried into the end of a soil specimen, it creates a vertically propagating, horizontally polarized shear wave. Since its electro-mechanical properties are interchangeable, the bender element can act as source and receiver.

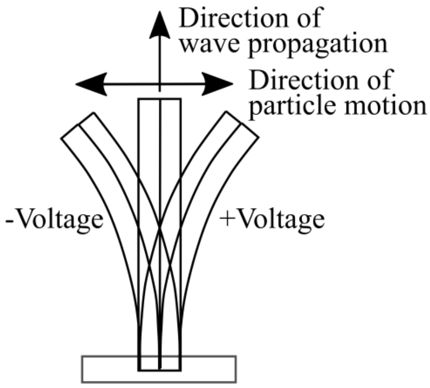

Fig.4 Bender element principle for shear wave transmission

Technical challenges of maintaining a dry device have been largely overcome and by carful fabrication, strong, clear, repeatable pulses can be created (Fig.5). In 2007, ISSMGE committee TC-29 issued a report on a set of tests performed by 30 different laboratories around the world [7]. Procedures for specimen preparation, measuring techniques, data processing and other parameters were examined and compared. The report identifies method of wave travel measurement and choice of signal and frequency as being important for obtaining good BE results.

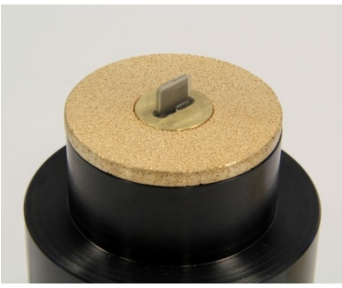

Fig.5 Bender element on triaxial end platen

6. APPLICATION OF THE THREE TESTS

The soil selected for testing is a Danube River sand. It has the typical behavior of a fine sand with some silt and mica. Specimens were formed by pluviation in air for the RC-TOSS tests and by dry vibration on a small platform for the BE tests. Density, void ratio, and water content were varied for the BE testing [8]. However for comparisons, only a select few of the BE tests performed will be used. Table 1 lists the test conditions relevant to the comparison. The bender element tests were performed on a single specimen, confinement ranged from σ_3=25-500kPa. While the sample was confined, static vertical stress (σ_1) was applied, depending on the confining stress, typically in 3-4 stages.

Table 1 Test matrix for BE, RC, and TOSS tests

Test	σ_0 (kPa)	Load Stages	Comments
BE	25-500	38	Different σ_1-σ_3
TOSS	100	7	τ =10-65 kPa
RC	100	12	After TOSS (τ =50-65kPa)

Note: σ_0 = average confining stress

Travel time was measured and values for shear wave velocity, and shear modulus were computed. At each stage, shear wave measurements were performed using the bender elements. TOSS testing was performed at a single confining stress (100kPa) with 7 stages of applied shear stress (10-65kPa) for 200 cycles of loading. Following each stage, a low-amplitude RC test was performed. After the 50- and 65-kPa tests, higher amplitude RC tests were performed to evaluate modulus degradation and damping.

7. TEST RESULTS

The BE tests illustrated the influence of confining stress on low-amplitude shear modulus. Shown in Fig.6 is a plot of G vs. σ_1 and σ_3. The data are fit with an exponential equation with four parameters; A, B, C, D, such that:

$$G = A\sigma_3^B + C\sigma_1^D$$
$$= 11.53\sigma_3^{0.168} + 0.155\sigma_1^{0.903} \qquad R^2 = 0.86 \tag{6}$$

Open circles in Fig.6 are measured G values while filled circles are computed values using σ_1 and σ_3 identical to test conditions. The filled circles lie on the curved surface (Eq. 6) and the vertical lines are to help located matched pairs of measured/computed points. In some places it is difficult to see the measured points since the computed one lies on top of the measured point. The regression fit is good (R^2= 0.86), however we are investigating ways to improve our predictions.

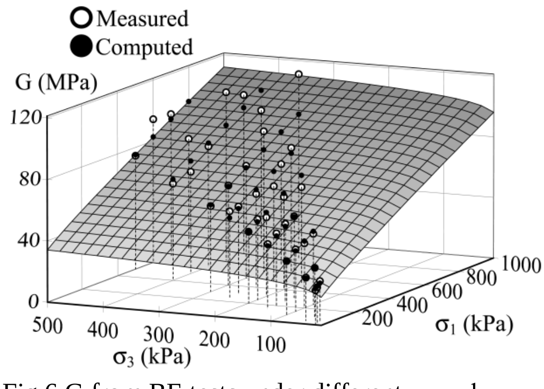

Fig.6 G from BE tests under different σ_1 and σ_3

The resonant column and torsional shear data were very much in agreement as shown in Fig.7. The secant shear modulus is on the left y-axis while damping is on the right. Shear strain is plotted logarithmically on the x-axis. Modulus reduction to less than 50% of maximum G (G_{max}) is seen at strain levels of about γ=0.1%. Damping ratio at this strain ranges between 8-12%. RC and TOSS tests were performed on the same specimen. Results from two specimens are shown in the figure. The TOSS data was obtained by measuring the end-to-end values of the stress-strain loop for the 10th and 200th cycle as well as the area of the hysteresis loop for damping. The RC data was obtained after six stages of torsional shear testing, so it should show more stiffness due to cyclic hardening during the tests. This is demonstrated by slightly higher G values and significantly lower D values. Other test data for Danube Sand show similar agreement [9].

Strain effects can be seen clearly in Fig.8 where a series of TOSS stress stages were applied. Shear stress, τ= 10, 15, 20, 25…50kPa was applied at each stage for 200 cycles with a low-amplitude RC (γ~10-4%) test in between to check for changes in G_{max}. As shown in Fig.8, the stiffness and damping change progressively for each stage and follow the backbone loading curve closely. Loading at τ=55kPa pushed the specimen beyond measurement limits and ended the test. We could fit the G vs. γ data (Fig.7) with a nonlinear model such as Ramberg-Osgood [10] and apply it to the hysteresis loops (Fig.8) using the Masing criteria. Effects of cyclic hardening or softening are more difficult to model. We are now testing specimens to see if it is better to change G_{max} or the Masing term. The modulus reduction and damping curves can be fit with other soil models including Vucetic-Dobry, and Hyperbolic [11,12]. These data are still in a relatively low strain range when compared to other TOSS data [13,14]. A full discussion of models, data fitting, and application to site response analysis or other geotechnical models such as t-z curves are beyond the scope of this paper.

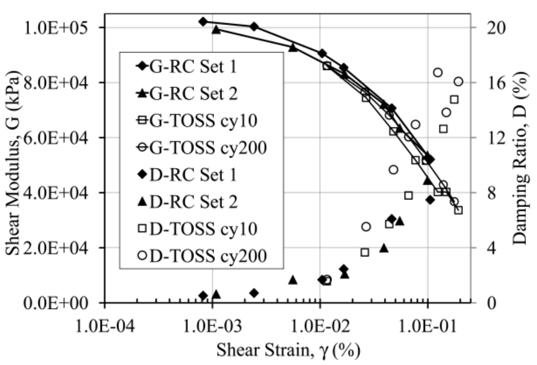

Fig.7 RC and TOSS test results

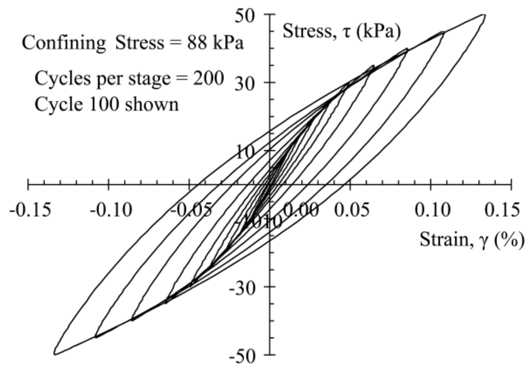

Fig.8 TOSS stress strain hysteresis loops for nine progressively higher cyclic stress stages

The data from BE and RC-TOSS tests share common conditions when the shear strain level is very low and confinement is (relatively) isotropic. A comparison of stress states of the tests are shown in Fig.9. Starting with a specimen confined isotropically (Mohr circle would be a point) or with some K_0 loading (small circle in Fig.9). As the specimen is loaded by the RC-TOSS device, the circle gets larger, but keeps the same center. For triaxial tests, the minor principle stress remains constant and the circle expands to the right. The shear wave motion and travel are opposite extremes for the two types of tests. Particle motion for RC-TOSS shear wave is parallel to the plane of maximum shear while for the BE, particle motion is parallel to the major principle plane (plane of zero shear stress). We believe that this difference is worth researching further since it addresses effects of stress anisotropy. Values of shear modulus for the different tests at similar confinement are shown in Fig.10. The RC and TOSS data were performed at a single confining stress at different strain levels. The data for the RC and TOSS tests would actually plot on top of each other at 100kPa average confining stress. While the data are encouraging, the results generate more questions than answers. Specifically, why don't the data agree more closely? The authors feel that some parameters are not yet accounted for, such as strain levels in the triaxial confinement process and the orientation of stress state and wave direction.

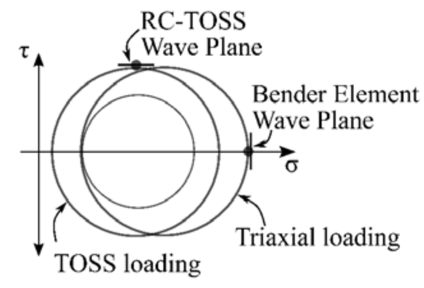

Fig.9 Stress states and wave particle motion

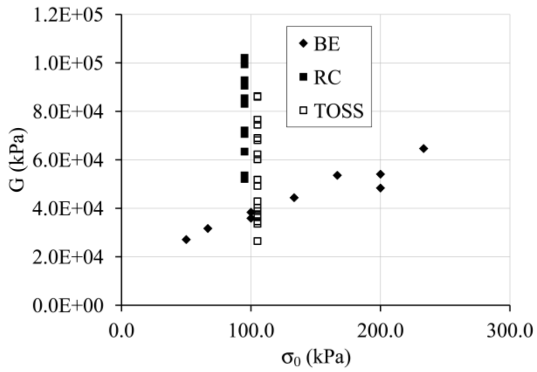

Fig.10 Comparison of shear modulus values obtained from different test methods

Other factors may be due to bender element coupling with the soil [15]. Further research will determine the influence of these factors. Results from the TC-29 parallel testing program [7] pointed out similar difficulties in obtaining consistent results with BE tests. More precise definition of source and receiver signals, improving the sampling interval (not too short or too long), and choosing an optimum set of frequencies to perform multiple measurements may improve consistency and compatibility with the other test results.

8. CONCLUSIONS

Danube River Sand was tested by three methods to determine its shear modulus under various conditions. Each method allowed for specific parameters to be studied and some were found to behave in a predictable manner. Resonant column and Torsional Simple Shear tests produced nearly identical properties for shear modulus and damping at various shear strain levels.

Bender element tests demonstrated the influence of stress anisotropy on shear modulus, but we were less successful correlating the results with the other two tests. Ranges of shear modulus values are similar, but not as close as expected. Additional research will be directed toward reconciling the differences.

9. ACKNOWLEDGEMENTS

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Corresponding Author: Zsolt Szilvágyi