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SLOVAK UNIVERSITY OF
TECHNOLOGY IN BRATISLAVA
FACULTY OF CIVIL ENGINEERING

**Slovenská technická univerzita v Bratislave**
**Stavebnej fakulty**
**Katedra geotechniky**

**Zborník 12. Slovenskej geotechnickej konferencie**

**Proceedings of the 12th Slovak Geotechnical Conference**

# 55 ROKOV GEOTECHNIKY NA SLOVENSKU

## 55 YEARS GEOTECHNICAL ENGINEERING IN SLOVAKIA

1. a 2. júna 2015, Bratislava, Slovenská republika

Zborník príspevkov 11. Slovenskej geotechnickej konferencie
**55 ROKOV GEOTECHNIKY NA SLOVENSKU**
Vydala Slovenská technická univerzita v Bratislave, Katedra geotechniky
Radlinského 11, 813 68 Bratislava, Slovenská republika.
Náklad 100 kusov CD, 505 strán.
Editor:  Monika Súľovská

Za obsahovú stránku príspevkov zodpovedajú autori.
Príspevky neprešli jazykovou úpravou.
Recenzovali: prof. Ing. Peter Turček, PhD., doc. Ing. Jana Frankovská, PhD.,
doc. RNDr. Miloš Kopecký, PhD.

**ISBN 978-80-227-4363-1**

Proceedings of the 12th Slovak Geotechnical Conference
**55 YEARS GEOTECHNICAL ENGINEERING IN SLOVAKIA**
Published by Slovak University of Technology, Faculty of Civil Engineering,
Radlinského 11, 813 68 Bratislava, Slovak republic.
The texts of all papers in this proceeding were prepared individually by their authors
and were rewieved.

**ISBN 978-80-227-4363-1**

*55 rokov geotechniky na Slovensku, Bratislava 01.- 02. júna 2015*

# DETERMINATION OF SOIL MODULUS BY THREE INDEPENDENT METHODS

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## ABSTRACT

This study compares results from three different testing methods: Resonant Column, Torsional Simple Shear, and Bender Element tests. The resonant column and torsional shear tests were performed on the same hollow cylinder specimen. The bender element test was performed on a triaxial specimen with the same void ratio and confining stress as well as others.

Several effects were studied, among them confining stress, shear strain amplitude and for the bender element, anisotropic confinement. Testing methods and data analysis are discussed in the paper because data interpretation is very important in these tests. Results showed that the shear modulus values were almost identical between the resonant column and torsional shear by varied somewhat with the bender element results laboratory tests and application of these results to static as well as dynamic analysis and design are discussed.

## 1. Introduction

Soil modulus has been used to model the low and medium stress-strain behavior of soil since the development of soil mechanics. Using elasticity theory and judiciously selected values for modulus and Poisson's ratio, geotechnical engineers have estimated foundation settlement, pile load deflection behavior in both vertical and horizontal directions and a wide variety of other applications. One of the difficulties with such an approach is the inability to make accurate and repeatable measurements of soil modulus. Measuring Young's modulus E, or shear modulus G in the field or laboratory is a challenge. For most applications, the measured modulus is back calculated from a fairly complicated test condition: a penetrating cone tip or sliding sleeve, or an expanding cylinder within a boring. In the laboratory there are also problems with precision, sample size and uniform stress or strain state. While certain properties such as void ratio, water content, or density can be faithfully repeated, fabric, cementation, and particle orientation can not. One of the difficulties associated with laboratory tests is achieving a low enough strain level that will replicate field conditions. Strain values on the order of $10^{-4}$ % are necessary to achieve a truly low-strain elastic condition. This paper discusses measurement of elastic modulus at low strains ($10^{-4}$ to 0.1 %) by three different methods: Resonant column, torsional shear, and bender element. Each test method has advantages and drawbacks. The best situation would be to use all three if necessary, but how would one know they are measuring the same property? Laboratory tests on nearly identical samples of Danube sand were performed by each of the three methods and the results were examined to see if they could be used interchangeably.

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## 2. Resonant Column Torsional Simple Shear (RC-TOSS) Device

The RC-TOSS device can measure rotational displacement and acceleration, longitudinal displacement, torque and pore pressure. These are accomplished by proximitors, an accelerometer, an LVDT, by coil current, and a pressure transducer, respectively. For resonant column tests, resonant frequency and acceleration determine modulus and strain level. For TOSS tests, the current flowing through the drive coils provide torque (stress) and proximitors measure rotation (strain) of the free end. A function generator provides the harmonic signal for RC testing while DC voltage from the data acquisition drives the TOSS test. Both tests require a high power amplifier (500 watts) to provide higher torque levels (about 50 N-m) for nonlinear conditions.

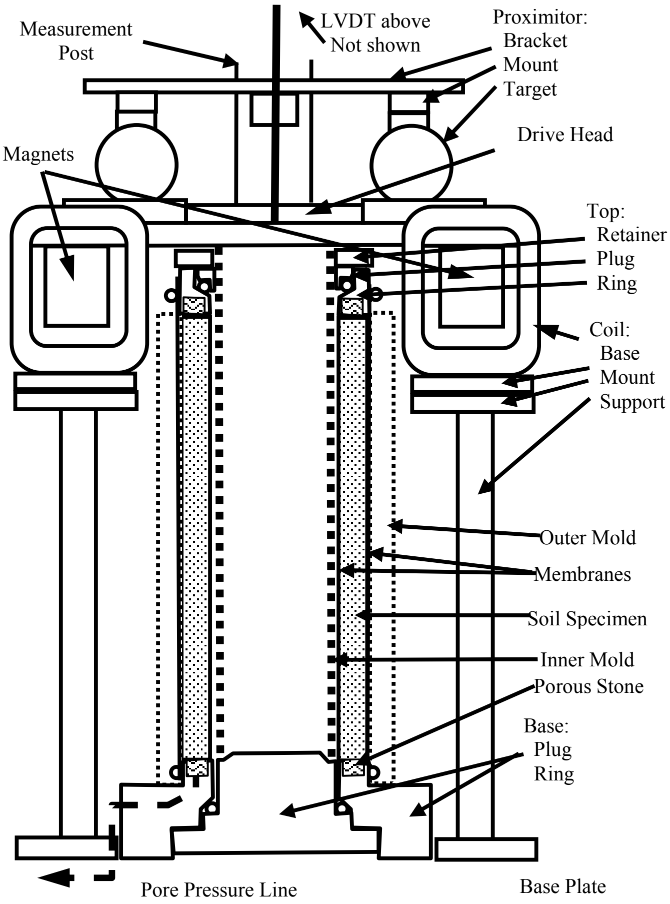


Fig. 1. Cross section of hollow cylinder Resonant Column / Torsional Shear device and specimen. Specimen is 4cm ID, 6cm OD and 14 cm high. Magnets are free to move through coils without touching. Base is fixed, top is free to twist.

By using an accelerometer, resonant column measurements benefit from the harmonic relationship between acceleration and displacement. This makes measurements at strain amplitudes near 10-4 % routine. A simple computation illustrates the concept. Typical accelerometer sensitivity is 300 mV/g, and a reading of 5 mV RMS (7.07 mV peak) from a good digital meter is common. The final necessary reading is resonant frequency, for the configuration in figure 1, a resonant frequency $\omega_n$=375 rad/sec.

Acceleration = (7.07/300) (9.807) (100) = 23.1 cm/sec^2
Displacement = $23.1/\omega^2 = 23.1/(375)^2 = 1.64\mathrm{x}10^{-4}$ cm
Peak Strain = displ(avg radius/radial distance accel)/length
= $1.64\mathrm{x}10^{-4}$ (2.5/4.70)/14.0=6.244e-6 mm/mm or 6.24e-4 %

The measurement is simple, accurate and repeatable. This is perhaps the greatest advantage of RC testing. For TOSS tests, the proximitors are used so that nothing touches the free end of the specimen. This allows for switching between TOSS and RC at any time. Proximitors measure the air gap between a steel targets mounted on the specimen drive head and the tip of the proximitor. Typical gap distance is 2 mm and sensor repeatability allows for accuracy to about 0.001 mm. This translates to shear strain levels of about $10^{-2}$ % to 1 %.

Vertical movement is monitored by an LVDT with accuracy to about 0.01 mm. The height change can often indicate changes in density or dilatent/contractile behavior. Pore water volume/pressure can also be monitored, however samples for these tests were dry or well below saturation level.

## 3. Resonant Column Test Method

The procedure for testing the soils by Resonant Column method generally followed that outlined in ASTM D 4015-07, Standard Test Methods for Modulus and Damping of Soils by the Resonant Column Method (ASTM 2007). Due to the unique nature of the material and the focus on high-amplitude behavior, the procedure deviated slightly from ASTM standards at several points listed below:

1. The specimens used were hollow cylinders. This allowed for more uniform torsional strain distribution, hence more representative values of G and D at high strain levels.
2. Platens had a sintered bronze filter stone rigidly bolted to the end caps to maintain full contact between the specimen and device.
3. The bottom end was tightly fixed to the base of the device, yielding a virtually infinite torsional stiffness at the passive end.
4. The weight of the active end was counter-balanced with a spring and thin wire, generating only negligible torsional resistance. This produced a free-end on the active side.
5. Given the fixed-free system, the governing test equation reduced to:

$$v_s = \frac{\omega_n l}{\beta} \qquad \text{(Eq. 1)}$$

where, $v_s$ = shear wave velocity, $l$ = length of specimen, $\omega_n$ = resonant frequency, $\beta$ = device/specimen constant, computed from device and specimen mass and geometries. It is the solution to the implicit equation

$$\frac{J}{J_0} = \beta \tan \beta \qquad \text{(Eq. 2)}$$

and, $J$ = mass polar moment of inertia of specimen (computed at time of testing) $J_o$ = mass polar moment of inertia of device head (measured via torsional pendulum calibration)

Since all values in equations 1 and 2 are known before resonating, except the natural frequency and shear wave velocity, the evaluation requires only a simple spreadsheet equation. Damping for the RC test is evaluated using the log decrement method where amplitudes of two consecutive acceleration peaks are measured and damping is computed as

$$\delta = \ln\left(\frac{u_n}{u_{n+1}}\right) \quad and \quad D = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}} \qquad \text{(Eq. 3)}$$

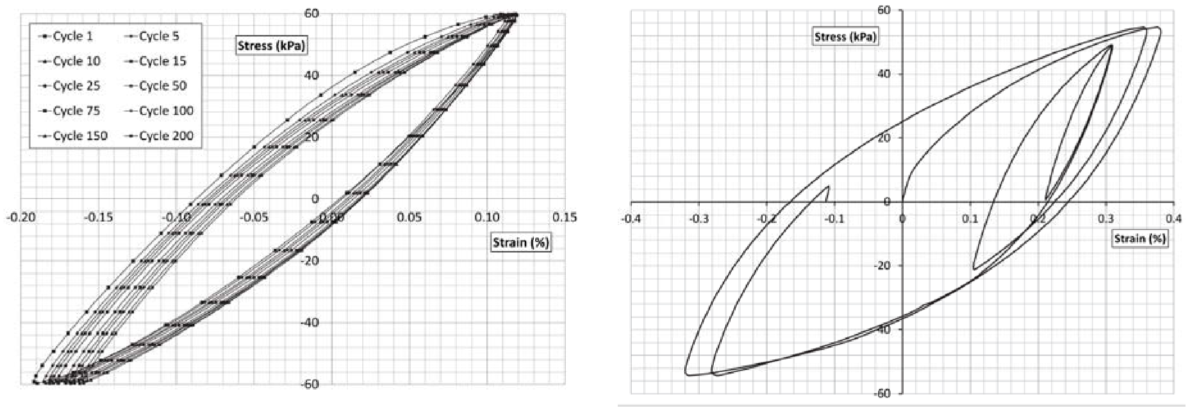
Where δ is log decrement, u is the peak amplitude of acceleration response of the *n* and *n+1* peaks and D is damping ratio. When expressed as a percentage, the value of D is multiplied by 100 %.

## 4. Torsional Shear Test Method

Using the same specimen dimensions, and often the same specimen, torsional shear testing is performed by driving the magnet coil system with a DC signal from the data acquisition card on the computer. The driving signal is a 16-bit digital-to-analog signal with peak output of ±10 volts DC. One step of the output corresponds to 0.0003 volts, so the resolution without any additional linear circuits is very good. The testing program is written in Visual Basic for Applications (VBA) which is free software in EXCEL. Since the data input and output is based on spreadsheets, the VBA approach is quite direct and simple. The only complication is making sure the data acquisition system is compatible and has the proper driver software for VBA. The torsional shear test control program follows theses steps:

1. Input all relevant specimen data (same as resonant column test)
2. Decide on cyclic or irregular loading history
    - 2a. Cyclic load determined by amplitude and offset. Maximum number of testing cycles is 65,000. Maximum number of cycles to record for data = 200, with 200 data points per recorded cycle.
    - 2b. Any arbitrary load history can be used, up to 10,000 points. Amplitude and offset of loading history can be set. All points are recorded as data.
3. Check readings from proximitors, LVDT, confinement pressure and make adjustments if necessary to make sure the sensors stay within their measuring range during testing.
4. Perform test. Program looks for next point in load history, if point is more positive than present point, the program drives one step in the positive direction. If the next point is more negative, the program drives one step in the negative direction.
5. At each driving step (4), the program checks data coming in
    - 5a. For cyclic loading if the data value reaches a data control point (one of the 200 each cycle) then the data for applied torque, rotation, vertical deflection, and confining pressure are recorded. And the next data control point is loaded for checking.
    - 5b. For irregular tests, all data is recorded for each point.
6. The test stops when all cycles (or irregular history points) are complete. A test of 200 cycles duration takes about 5 minutes to complete. An irregular history with 2000 data points requires about 1 minute or less.

The data is then output to the spreadsheet. Typical output for a cyclic test and irregular history test are shown in figure 2. Specimens were confined under vacuum at 90 kPa.



(a) (b)

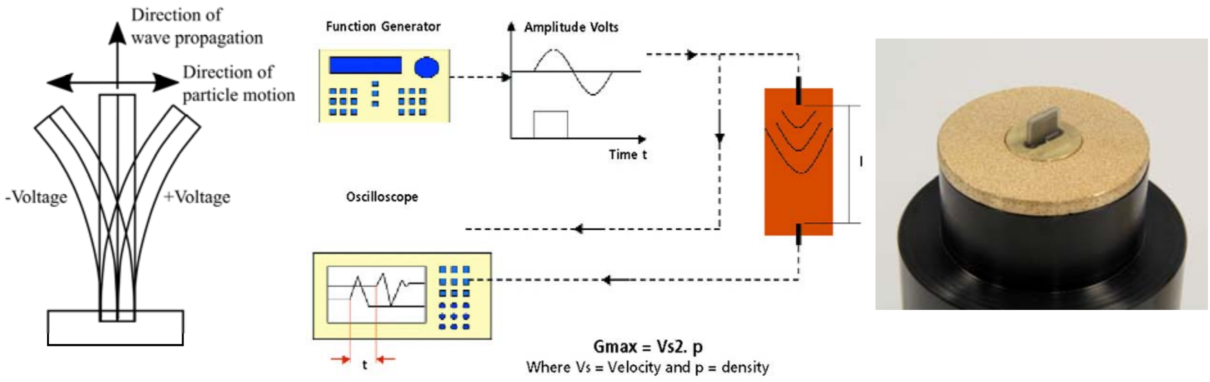
Fig. 2. Cyclic test results (a) for 200 uniform load cycles, note slight hardening with cycles; irregular test results (b) show large initial cycle with smaller reversals within, then back out to large cycle. Masing criteria clearly shown were reversals follow older loops when they reach end of inner (newer) loops.

For cyclic tests, secant shear modulus, $G_{sec}$ and hysteretic damping ratio $D_{hys}$ can be computed as well. Secant modulus is the slope of a line connecting to two ends of any loop. Hysteretic damping is computed from the area inside the stress-strain loop and the amount of work done to the specimen, for a uniform cycle of stress and strain with peak values of ($\pm\tau$, $\pm\gamma$):

$$D_{hys}(\%) = 100\frac{Area\ of\ loop}{2\pi\tau\gamma} \qquad \text{( Eq. 4)}$$

## 5. Bender Element Triaxial Test

Bender elements were developed around the late 1980’s and have improved steadily since that time. They consist of a lamination of Polyvinylidyne Flouride (PVDF), a polymer that has excellent piezo-electro-mechanical properties. The polymer film has a piezoelectric property much like quartz in that if the polymer is mechanically moved, the charge regime on it changes. Likewise if the charge regime changes, the polymer moves. Variations of this polymer are used as acoustic speakers for “earbud” headphones and other small-scale speakers. When fabricated as a laminated cantilever beam, and driven by a sine-wave voltage it will oscillate in first mode indefinitely (figure 3). When buried into the end of a soil specimen, it will create a vertically propagating, horizontally polarized shear wave. Since the electro-mechanical properties are interchangeable, the bender element can act as source and receiver. Technical challenges of maintaining a dry device have been largely overcome and by carful fabrication, strong, clear, repeatable pulses can be created. In 2007, ISSMGE committee TC-29 issued a report on a set of tests performed by 30 different laboratories around the world (TC-29, 2007). Procedures for specimen preparation, measuring techniques, data processing and other parameters were examined and compared.



(a) (b) (c)

Fig. 3. Bender element priciple for shear wave transmission (a), typical laboratory set-up (b), bender element on triaxial end platen

## 6. Application of the Three Tests

The soil selected for testing is a Danube River sand. It was chosen because it has typical behavior one would expect from a fine sand with some silt and mica, without any unusual properties or behavior. Specimens were formed by pluviation in air for the RC-TOSS tests and by dry vibration on a small platform for the triaxial tests. Density, void ration, and water content were varied for the triaxial testing. However for comparisons, only a select few of the tests performed will be used. The table below lists the test conditions relevant to the comparison

Table 1. Testing matrix for bender element, resonant column and torsional simple shear tests

| Test | Confinement (kPa) | Stress/Strain Stages | Comments |
|---|---|---|---|
| Bender Element | 25-500 | 38 | Different $\sigma_1$-$\sigma_3$ |
| Resonant Column | 100 | 12 | After TOSS 50, 60kPa |
| Torsional Simple Shear | 100 | 7 | 10-65 kPa |

The bender element tests were performed on a single specimen, confinement ranged from $\sigma_3$=25-500 kPa. While the sample was confined, various levels of vertical stress ($\sigma_1$) were applied depending on the confining stress, typically 3-4 stages. At each stage, shear wave measurements were performed using the bender elements. Travel time was measured and values for shear wave velocity, and shear modulus were computed.

RC-TOSS testing was performed at a single confining stress (100 kPa) with 7 stages of applied shear stress (10-65kPa) for 200 cycles of loading. Following each stage, a low-amplitude resonant column test was performed. After the 50- and 60-kPa tests, higher amplitude RC tests were performed to evaluate modulus degradation and damping.

## 7. Test Results

The bender element tests illustrated the influence of confining stress on low-amplitude shear modulus. Shown in figure 4 is a plot of G vs. average confining stress $\sigma_0$ for all tests. The data are fit with an exponential equation shown in the figure. Note that the horizontal axis has not functional relationship with the curves, but is a convenience to show the data together in a two

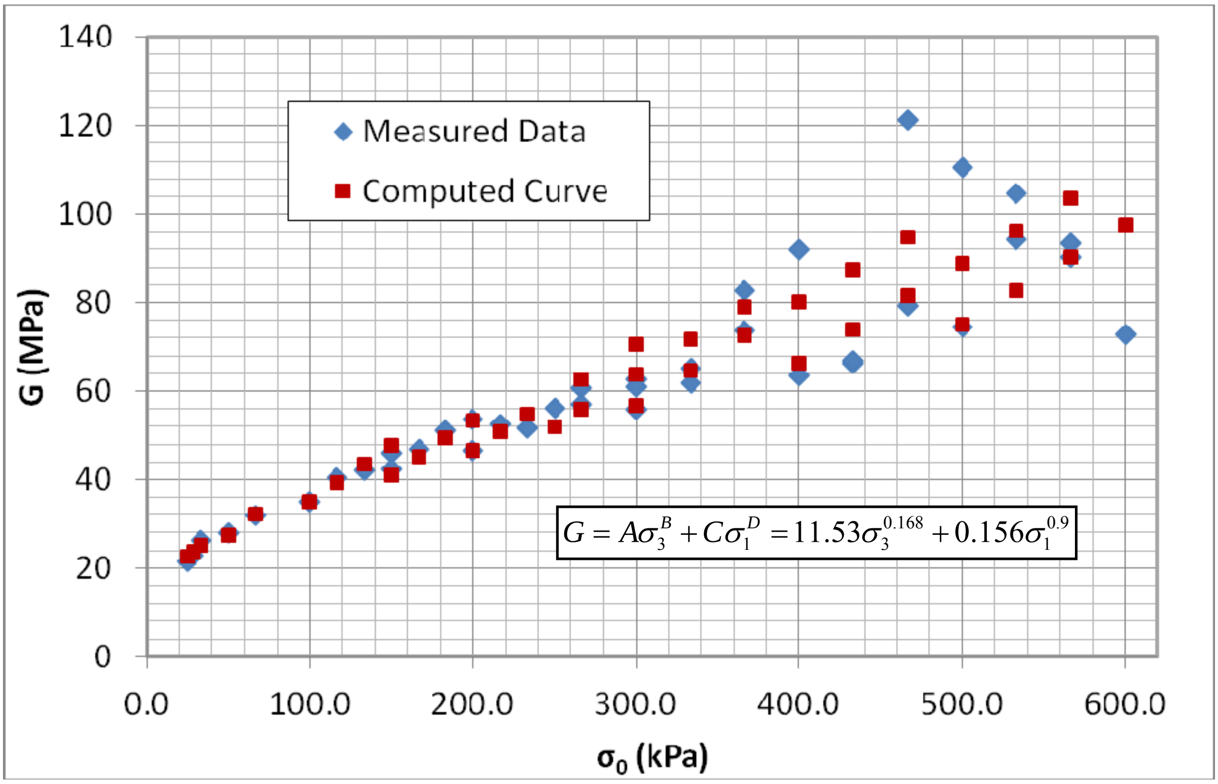


Fig. 4. Results from bender element test. The data is fit with an exponential equation for $\sigma_1$ and $\sigma_3$. The axis $\sigma_0$ is just to help show the data.

dimensional plot. The accuracy of the prediction is represented by the vertical distance between pairs of measured and computed data points. The fit is fair, however, we are investigating better ways to model the data, such as using other strain-based expressions.

The resonant column and torsional shear data agreed very well with each other. Shown in figure 5 is a plot of shear modulus vs. strain amplitude for TOSS and RC data on the same specimen. The RC data was obtained after six stages of torsional shear testing, so it should show more stiffness due to the cyclic hardening during the tests. Other test data for Danube Sand show similar agreement. The TOSS data was obtained by measuring the end-to-end values of the stress-strain loop for the 10th and 200th cycle as well as the area of the hysteresis loop for damping.

Another topic of research is examining alternative ways to compute damping in RC tests. The modulus reduction and damping curves can be fit with a variety of soil models including Vucetic-Dobry, Ramberg-Osgood, MIT-3, Darendeli, Hardin-Drnevich, and others. A full discussion of models, data fitting, and application to site response analysis or other geotechnical models such as t-z curves are beyond the scope of this paper. Evaluation of G and D for various conditions of shear strain, confining pressure, number of cycles of loading, and initial soil conditions are a continuing focus of research.

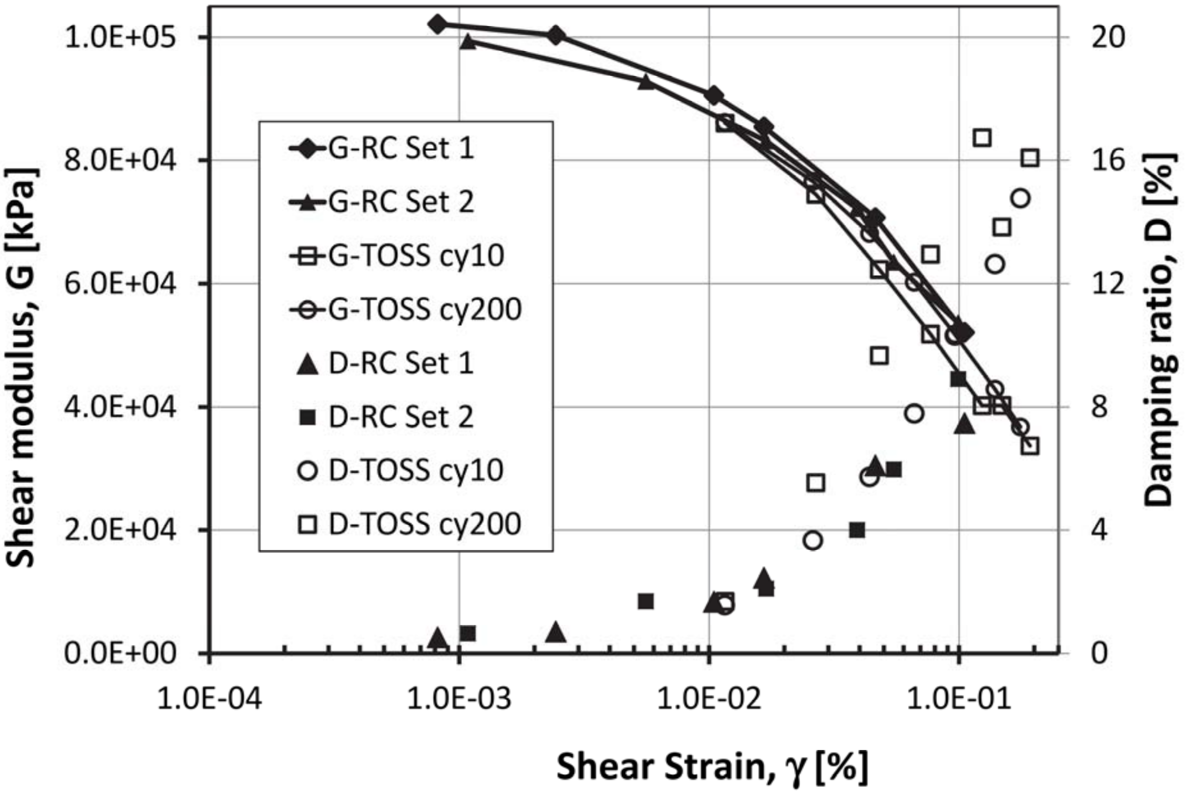


Fig. 5. Resonant Column and Torsional Simple Shear test results. Secant shear modulus vs. shear strain for both RC and TOSS tests. Damping values also shown from RC tests. Strain effect shows a stiffness reduction to 30% of original low-amplitude (field seismic) values

The data from bender element and RC-TOSS tests share common conditions when the shear strain level is very low, confinement is (relatively) isotropic. A comparison of stress states of the tests are shown in figure 6. Starting with a specimen confined isotropically (Mohr circle would be a point) or with some $K_0$ loading (small circle in figure 6).

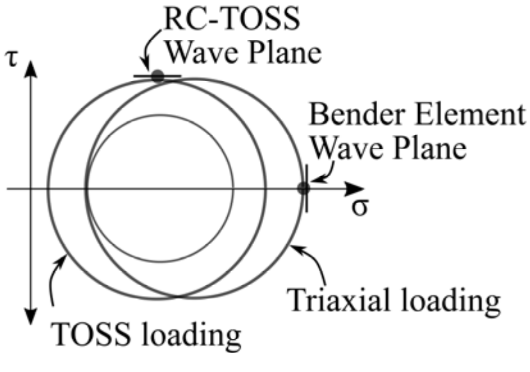


Fig. 6. Comparison of stress states and wave particle motion. Initial stress state shown as small circle (zero radius possible). RC-TOSS and Triaxial loading are different. RC-TOSS shear wave particle motion is parallel to maximum shear plane opposite is true for bender element triaxial.

As the specimen is loaded by the RC-TOSS device, the circle gets larger, but keeps the same center. For triaxial tests, the minor principle stress remains constant and the circle expands to the right. The shear wave motion and travel are opposite extremes for the two types of tests. Particle motion for RC-TOSS shear wave is parallel to the plane of maximum shear while for the triaxial bender element, particle motion is parallel to the major principle plane (plane of zero shear stress). We believe that this difference is worth researching further since it addresses effects of stress anisotropy.

Values of shear modulus for the different tests at similar confinement are shown in figure 7. The RC and TOSS data were performed at a single confining stress at different strain levels.

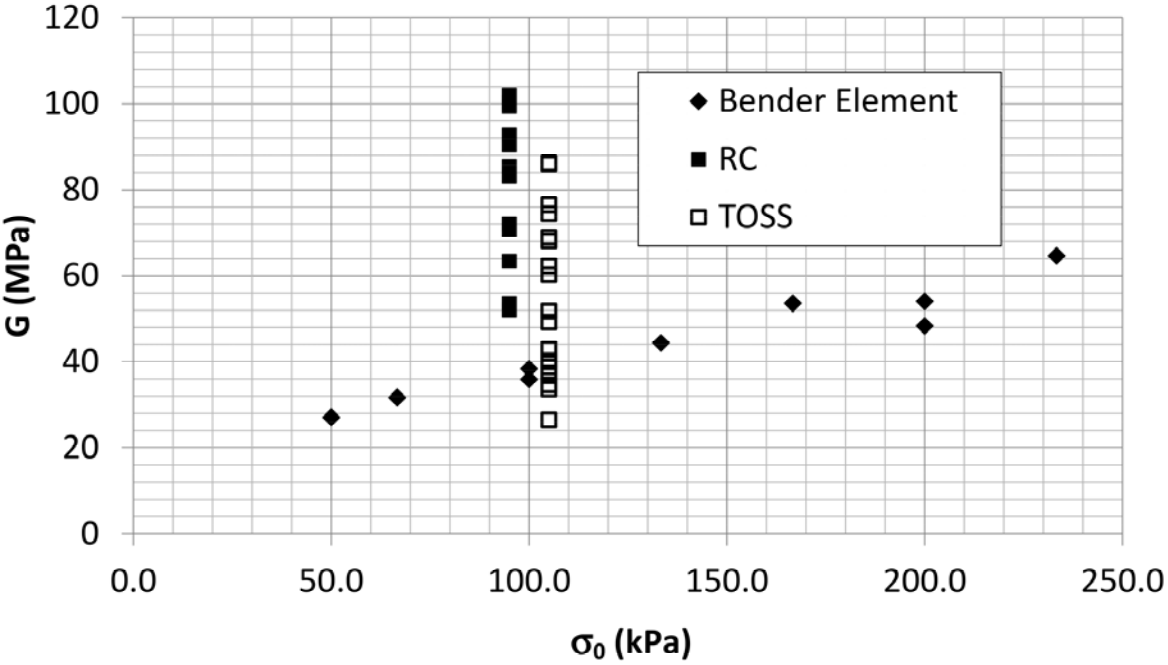


Fig. 7. Comparison of shear modulus values from different test methods.

The data for the RC and TOSS test would actually plot on top of each other at 100 kPa average confining stress. While the data are encouraging the results generate more questions than answers. Specifically, why don’t the data agree more closely? We feel that some parameters are not yet accounted for, specifically strain levels in the triaxial confinement process and the orientation of stress state and wave direction. Other factors may be due to bender element coupling with the soil. Further research will determine the influence of these factors.

## 8. Conclusions

Danube River Sand was tested by three methods to determine its shear modulus under various conditions. Each method allowed for specific parameters to be studied and some were found to behave in a predictable manner. Resonant column and Torsional Simple Shear tests produced nearly identical properties for shear modulus and damping at various shear strain levels.

Bender element tests demonstrated the influence of stress anisotropy on shear modulus, but we were less successful correlating the results with the other two tests. Ranges of shear modulus values are similar, but not as close as expected. Additional research will be directed toward reconciling the differences.

## References

TC-29 ISSMGE (2007) *International Parallel Test on the Measurement of Gmax Using Bender Elements Organized by TC-29*, 76p.